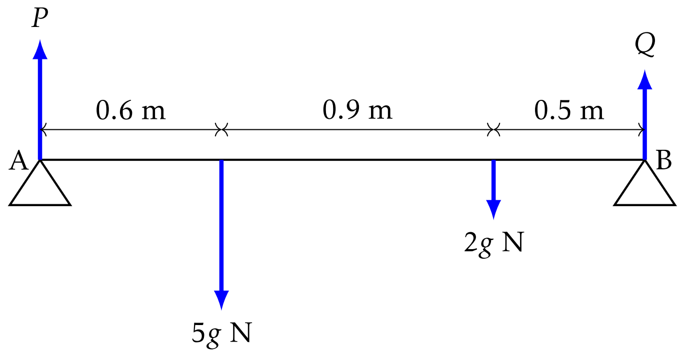
Revision

Example

Two forces, of magnitude $5g$ N and $2g$ N, are applied at points on a light rod supported by two pivots (A and B) as shown in the diagram. Find the reaction forces



$$N2(\uparrow, beam): \quad P - 5g - 2g + Q = 0$$
$$\Rightarrow \quad P + Q = 7g$$

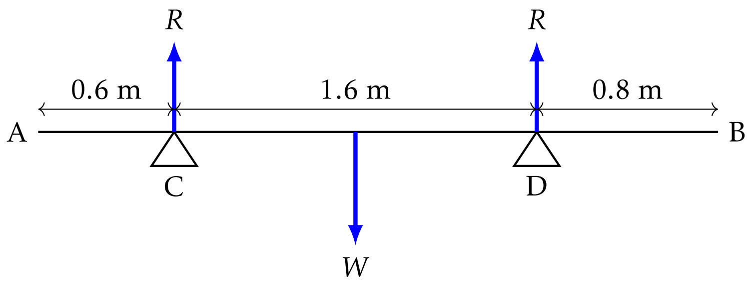
$$\overset{\curvearrowright}{B}: \quad P \cdot 2 - 5g \cdot 1.4 - 2g \cdot 0.5 = 0$$
$$\Rightarrow \quad 2P = 7g + g$$
$$\Rightarrow \quad P = 4g$$
$$\Rightarrow \quad Q = 3g$$

Tip (Pick a good place to take moments about!)

- Can you eliminate a force you don't know
- Can you exploit some symmetry in the diagram?

Example

A non-uniform plank of length 3 m rests in equilibrium on two supports, located at 60 cm and 80 cm from each end. The reaction forces in the two supports are equal. Find the position of the centre of mass of the plank.



$$N2(\uparrow, beam): \quad 2R - W = 0$$
$$\Rightarrow \quad R = \frac{1}{2}W$$

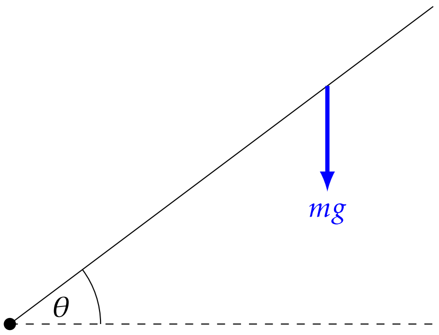
$$\overset{\curvearrowright}{C}: \quad W \cdot x - R \cdot 1.6 = 0$$
$$\Rightarrow \quad xW = \frac{1}{2}W \cdot 1.6$$
$$\Rightarrow \quad x = 0.8$$

Therefore the centre of mass is a distance 1.4 m *from the first end, or equidistant between the supports. Could we have seen this another way? What if we took moments about the centre of mass?*

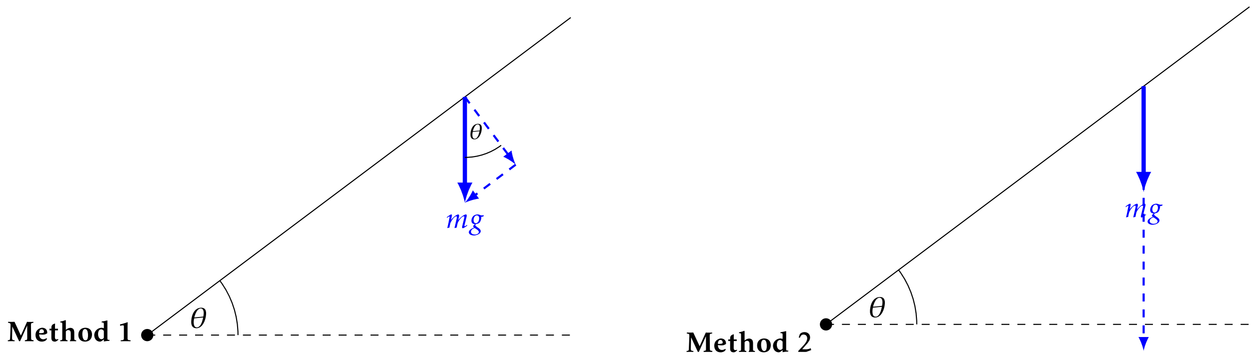
Moments with non-perpendicular forces

Example

Suppose we have a force acting on a rod, at a distance 1 m from a pivot. What is the moment of the force about the pivot?

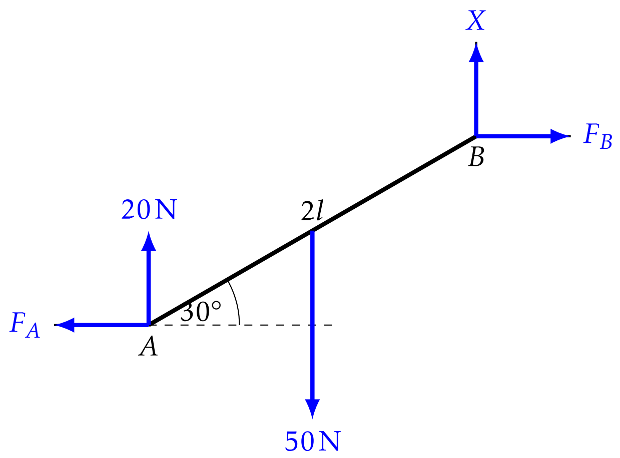


Fact — The *moment* of a force F about a point P is the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the point P.



Example

A uniform rod of length $2l$ and weight 50 N is held in equilibrium by four light inextensible strings, as shown in the diagram. Two of the strings are horizontal and two are vertical. The rod makes a 30° angle with the horizontal. The tension in the vertical string to the left of the diagram is 20 N. Find the tensions in the two horizontal strings.



$$N2(\rightarrow): \quad F_B - F_A = 0$$
$$\Rightarrow \quad F_A = F_B$$

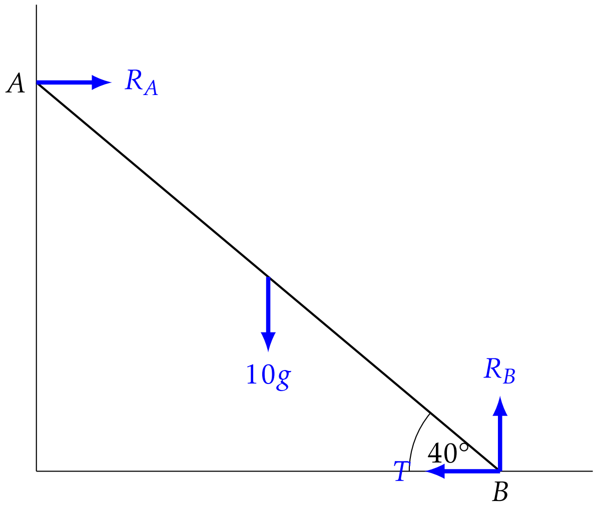
$$\overset{\curvearrowright}{B}: \quad 20 \cdot 2l\cos(30^\circ) + F_A \cdot 2l\sin(30^\circ) - 50 \cdot l\cos(30^\circ) = 0$$
$$\Rightarrow \quad F_A = (50 - 40)\cos(30^\circ)$$
$$= 5\sqrt{3}\,N$$
$$= 8.66\,N\ (3\,s.f.)$$

Ladders

Example

A uniform ladder AB, of mass 10 kg and length 4 m, rests with its upper end A against a smooth vertical wall and end B on smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium inclined at 40° to the horizontal. The vertical plane containing the ladder and the string is at right angles to the wall.

Find the tension T in the string and the normal reactions at the points A and B.



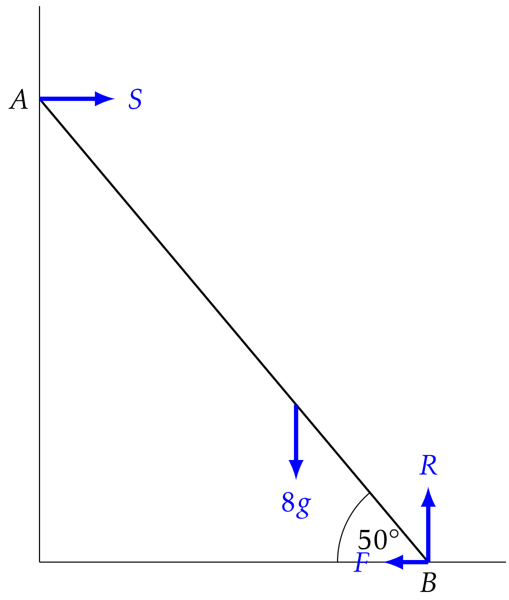
$$N2(\uparrow): \quad -10g + R_B = 0$$
$$\Rightarrow \quad R_B = 10g$$

$$N2(\rightarrow): \quad R_A - T = 0$$
$$\Rightarrow \quad R_A = T$$

$$\overset{\curvearrowright}{B}: \quad R_A \cdot 10\sin(40^\circ) - 10g \cdot 5\cos(40^\circ) = 0$$
$$\Rightarrow \quad R_A = 5g\cot 40^\circ$$
$$= 58.4\,\text{N}$$
$$\Rightarrow \quad T = 58.4\,\text{N}$$

Example

The diagram shows a ladder AB of mass 8 kg and length 6 m resting in equilibrium at an angle of 50° to the horizontal with its upper end A against a smooth vertical wall and its lower end B on rough horizontal ground, coefficient of friction μ. Find the forces S, F and R and the least possible value of μ if the centre of gravity G of the ladder is 2 m from B.



$$\begin{aligned}
N2(\uparrow): \quad & -8g + R = 0 \\
\Rightarrow \quad & R = 8g \\
N2(\rightarrow): \quad & S - F = 0 \\
\Rightarrow \quad & S = F
\end{aligned}$$

$$\begin{aligned}
\overset{\frown}{B}: \quad & S \cdot 6\sin(50°) - 8g \cdot 2\cos(50°) = 0 \\
\Rightarrow \quad & S = \tfrac{8}{3} g \cot(50°)
\end{aligned}$$

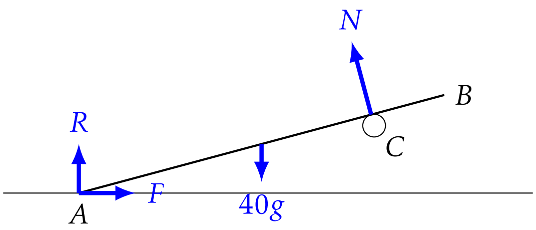
$$\begin{aligned}
\Rightarrow \quad \frac{F}{R} &= \frac{S}{R} \\
&= \frac{\frac{8}{3} g \cot 50°}{8g} \\
&= \tfrac{1}{3}\cot(50°) \\
&= 0.2796\ldots
\end{aligned}$$

Since $\mu = \frac{F_{max}}{R}$ *and* $\frac{F}{R} = 0.28$ *we can conclude* $\mu \geq 0.28$ *and so the least possible value for* μ *is* 0.28.

Example

A uniform rod AB of mass 40 kg and length 10 m rests with the end A on rough horizontal ground. The rod rests against a smooth peg C where $AC = 8$ m. The rod is in limiting equilibrium at an angle of 15° to the horizontal. Find

- the magnitude of the reaction at C,
- the coefficient of friction between the rod and the ground.



It is important first to notice that the force being applied by the peg on the rod acts perpendicular to the rod. (Since it rests as a tangent).

$$\overset{\frown}{A}: \quad -8N + 40g \cdot (5\cos 15^\circ) = 0$$
$$\Rightarrow \quad N = 25\cos(15^\circ)g$$
$$= 236.65\ldots \text{N}$$

So the reaction at C is 237 N (3 s.f.).

$$N2(\rightarrow): \quad F - N\sin(15^\circ) = 0$$
$$\Rightarrow \quad F = 61.25\ldots \text{N}$$
$$N2(\uparrow): \quad R - 40g + N\cos(15^\circ) = 0$$
$$\Rightarrow \quad R = 40g - N\cos(15^\circ)$$
$$= 163.4\ldots \text{N}$$

Since we are in limiting equilibrium, $F = \mu R \Rightarrow \mu = 0.37$